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Instability of the one-dimensional extended Hubbard model in a magnetic field

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Abstract

We consider instabilities of two-component Tomonaga–Luttinger (TL) liquids for the one-dimensional extended Hubbard model in a magnetic field. When the density of electrons with spin σ is $1/2$, electrons with spin σ tend to occupy every other site, and do not conduct. The instability stems from the umklapp scattering between same-spin electrons. But for the remaining electrons with spin $-\sigma$, an energy gap does not appear. We also consider the instability relating to the phase separation. This relates to the intrinsic stability condition of the TL liquid.

1. Introduction

For several one-dimensional (1D) quantum systems, the long wavelength behaviour is described by the Tomonaga–Luttinger (TL) liquid [1–9]. Elementary excitations are the sound wave relating to the density fluctuation and topological excitations. Correlation functions at $T = 0$ decay by a power law, and exponents of them change continuously with the strength of the interaction.

The 1D Hubbard model can be recognized as a two-component TL liquid. In zero external magnetic field this model has a $U(1) \times SU(2)$ symmetry. The $U(1)$ symmetry relates to the charge freedom and the $SU(2)$ to the spin one, and the velocities of collective sound wave excitations are different between these freedoms. Except for the half-filling case, properties of the two-component TL liquid remain at the strong coupling limit [10–14]. In a magnetic field, the symmetry of the model reduces to a $U(1) \times U(1)$ symmetry and we cannot expect the independent collective excitations relating to the charge and spin freedoms. Woynarovich [15] calculated the finite-size ground-state energy and excitation spectrum with a Bethe ansatz (BA) [16]. He analysed them with conformal field theory [17–20] and showed the tower structure of scaling dimensions in excitation energies and the central charge of the Virasoro algebra in the ground state energy. With a finite-size spectrum from the BA calculation and conformal field theory, Frahm and Korepin [14, 21] (see also [22]) derived the form of the correlation function and critical exponents. According to them [14, 15, 21], the low-energy behaviour for the non-half-filling case is that of two TL liquids. Ogata *et al* [23] calculated the momentum distribution function with a finite-size BA wavefunction for the large U/t limit. The position of the singularity of these functions is related to $k_{F\pm}$, as for the non-interacting case. Penc and Sólyom [24] studied the model in relation to the g-ology and calculated the correlation functions of electrons for the general multi-component case with a Ward–Takahashi identity.

In this paper, we study the instability of the 1D extended Hubbard model in a magnetic field. We can recognize the system as the (asymmetrically) coupled TL liquid. Due to the nearest neighbour interaction, two instabilities can appear. One instability stems from the umklapp scattering between electrons with the same spin. The other one relates to the phase separation. We consider these instabilities for coupled TL liquids.

The organization of this paper is as follows. In the next section, we present the model. In section 3, we present the effective model with the bosonization approach. In these two sections, we see that, in a magnetic field, the umklapp scattering with same-spin electrons can develop an excitation gap. Section 4 gives the correlation function of the phase field defined in section 3 for the free field case. In section 5, we consider the instability and properties of the system. The last section is devoted to a summary and discussion.

2. Model

The Hamiltonian of the 1D extended Hubbard model in a uniform magnetic field is given by

$$H = -t\sum_{\sigma=\pm}\sum_{j=1}^{L}(c^{\dagger}_{j,\sigma}c_{j+1,\sigma} + c^{\dagger}_{j+1,\sigma}c_{j,\sigma}) + U\sum_{j=1}^{L} n_{j,+}n_{j,-} + V\sum_{j=1}^{L}(n_{j,+}+n_{j,-})(n_{j+1,+}+n_{j+1,-}) - \frac{h}{2}\sum_{j=1}^{L}(n_{j,+}-n_{j,-}), \tag{1}$$

where L is the system size, $c_{j,\sigma}$ and $c^{\dagger}_{j,\sigma}$ are the annihilation and the creation operator of the electron at the jth site with spin $\sigma = \pm$, $n_{j,\sigma} = c^{\dagger}_{j,\sigma}c_{j,\sigma}$ is the number of electrons with spin σ at the jth site, U and V are the on-site and the nearest neighbour interactions and h is proportional to an external magnetic field. We assume the periodic boundary condition $c_{j+L,\sigma} = c_{j,\sigma}$. For $h = 0$, several studies have been done and phase diagrams for several electron densities were obtained [25–36].

After the Fourier transformation of the free part:

$$H_{\text{free}} = -t\sum_{\sigma=\pm}\sum_{j=1}^{L}(c^{\dagger}_{j,\sigma}c_{j+1,\sigma} + c^{\dagger}_{j+1,\sigma}c_{j,\sigma}) - \frac{h}{2}\sum_{j=1}^{L}(n_{j,+}-n_{j,-}),$$
$$c_{j,\sigma} = \frac{1}{\sqrt{L}}\sum_{k} \mathrm{e}^{\mathrm{i}kj}c_{k,\sigma}, \qquad k = -\pi + \frac{2\pi}{L}, -\pi + 2\frac{2\pi}{L}, \ldots, \pi, \tag{2}$$

we have

$$H_{\text{free}} = \sum_{\sigma=\pm}\sum_{k}\left[-2t\cos k c^{\dagger}_{k,\sigma}c_{k,\sigma} - \sigma\frac{h}{2}c^{\dagger}_{k,\sigma}c_{k,\sigma}\right]. \tag{3}$$

In a magnetic field, the Fermi wavenumber k_{F+} for the $\sigma = +$ spin electron and k_{F-} for the $\sigma = -$ spin electron are not the same. Thus the Fermi velocities v_{F+} for $\sigma = +$ fermion and v_{F-} for $\sigma = -$ are different.

On introducing the nearest neighbour interaction V, umklapp scattering between two electrons with the same spin σ appears. Due to the commensurability, this umklapp scattering process is important for the case where the filling of the electrons with spin σ is $n_\sigma = 1/2$, and there is a possibility of an instability appearing for the TL liquid. The instability is the same as the metal–insulator transition of the lattice spinless fermion at half filling ($\sum_j c_j^\dagger c_j = L/2$)

$$H = -t \sum_{j=1}^{L} (c_j^\dagger c_{j+1} + c_{j+1}^\dagger c_j) + V \sum_{j=1}^{L} n_j n_{j+1}. \tag{4}$$

For the weak coupling case $-2t < V < 2t$, the low energy physics is described by the TL liquid. But for the strong coupling case $2t < V$, fermions tend to occupy every other site and the system is an insulator. This instability of the TL liquid comes from the umklapp scattering [7]. For the extended Hubbard model (1), when $n_\sigma = 1/2$, it is expected that the same thing occurs for electrons with spin σ due to the nearest neighbour interaction. The possibility of this instability can be derived from the argument of Yamanaka *et al* [37] (see also [38]). Even for the TL liquid system, a Fermi wavenumber $k_{F\sigma}$ exists and depends on the density of fermions. The instability relates to the condition whether the momentum transfer of the umklapp scattering process between Fermi wavenumbers matches the reciprocal lattice vector or not.

Let us consider where this instability of the extended Hubbard model can occur. We define the electron density as $n = n_+ + n_-$ and the magnetization density m for $n < 1$ as $m = (n_+ - n_-)/(n_+ + n_-)$, and for $1 < n$ as $m = (n_+ - n_-)/(2 - n_+ - n_-)$. These magnetization densities are normalized to 1 for the saturation magnetization. In the region $1/2 < n < 1$, the density of electrons with $\sigma = +$ can be $n_+ = 1/2$. For this density, the magnetization density is written as

$$m = \frac{1-n}{n}. \tag{5}$$

In the region $1 < n < 3/2$, the density of electrons with $\sigma = -$ can be $n_- = 1/2$. In this case, the magnetization density is given by

$$m = \frac{n-1}{2-n}. \tag{6}$$

Thus for these magnetizations (5) and (6), the above-mentioned instability can occur. After a transformation $c_{j,\sigma} \to (-1)^{j-1} c_{j,-\sigma}^\dagger$, we have

$$n \to 2-n, \qquad m \to m, \tag{7}$$

and invariant U, V, h in the Hamiltonian. Thus two systems with densities (n, m) and $(2-n, m)$ show the same properties, and two lines (5) and (6) are symmetric for the line $n = 1$. (On these lines, we should exclude the point $n = 1$, $m = 0$, because another type of instability occurs at this point [25, 27, 28, 30, 31, 35, 36].)

Another instability of the one-component model (4), phase separation, appears in the region $V < -2t$. This comes from violation of the intrinsic stability condition of the TL liquid. This type of instability can also appear in the model (1).

3. Bosonization

In order to consider the instability, we use the bosonization approach. We linearize the dispersion of the single electron around the Fermi point and the electron operator is written as

$$c_{x,\sigma} = \sqrt{a}(\mathrm{e}^{-\mathrm{i}k_{F\sigma}x}\psi_{\mathrm{L}\sigma}(x) + \mathrm{e}^{\mathrm{i}k_{F\sigma}x}\psi_{\mathrm{R}\sigma}(x)), \tag{8}$$

where a is the short-range cut-off. The operator $\psi_{\mathrm{L}\sigma}(x)$ is the annihilation operator for the left-moving fermion and $\psi_{\mathrm{R}\sigma}(x)$ for the right-moving fermion with spin σ. Introducing the phase field ϕ_σ and θ_σ, which satisfy the relation $[\phi_\sigma(x), \theta_{\sigma'}(x')] = -\delta_{\sigma,\sigma'}\mathrm{i}(\pi/2)\,\mathrm{sgn}\,(x-x')$, fermion operators are written as [4, 5, 8]

$$\psi_{\mathrm{L}\sigma}(x) = \frac{1}{\sqrt{2\pi a}}\exp\left(\mathrm{i}\frac{1}{\sqrt{2}}\phi_\sigma + \mathrm{i}\sqrt{2}\theta_\sigma\right), \quad \psi_{\mathrm{R}\sigma}(x) = \frac{1}{\sqrt{2\pi a}}\exp\left(-\mathrm{i}\frac{1}{\sqrt{2}}\phi_\sigma + \mathrm{i}\sqrt{2}\theta_\sigma\right). \tag{9}$$

Then we have the following bosonized Hamiltonian:

$$H = H_0 - \sum_{\sigma=\pm} g_\sigma \int \frac{\mathrm{d}x}{2\pi}\cos(2\sqrt{2}\phi_\sigma - 2(\pi - 2k_{F\sigma})x - 2k_{F\sigma}) \tag{10}$$

where $g_\sigma = 4V/\pi a$. Here we neglected the backscattering term and the umklapp scattering term involving different spin electrons. (The former is crucial for generating a spin gap in zero magnetic field [39] and the latter is important for the metal–insulator transition at half-filling [40].)

The last term of equation (10) comes from the umklapp scattering between electrons with the same spin, and for its oscillating nature with x, it is important only for $k_\sigma = \pi/2$ ($n_\sigma = 1/2$). The first term of equation (10) is given by

$$H_0 = \int \frac{\mathrm{d}x}{2\pi}\sum_{\sigma,\sigma'=\pm}\left[v_{0\sigma\sigma'}(\pi\Pi_\sigma)(\pi\Pi_{\sigma'}) + v_{1\sigma\sigma'}\left(\frac{\partial\phi_\sigma}{\partial x}\right)\left(\frac{\partial\phi_{\sigma'}}{\partial x}\right)\right], \tag{11}$$

where Π_σ is the momentum density conjugate to ϕ_σ, $[\phi_\sigma(x), \Pi_{\sigma'}(x')] = \mathrm{i}\,\delta_{\sigma\sigma'}\delta(x-x')$, and this is given by $\pi\Pi_\sigma = \partial\theta_\sigma/\partial x$. $v_{0\sigma\sigma'}$ and $v_{1\sigma\sigma'}$ have the dimension of the velocity and are elements of the following symmetric matrix:

$$\mathrm{v}_0 = \begin{bmatrix} 2v_{F+} + \frac{2Va\cos 2k_{F+}}{\pi} & 0 \\ 0 & 2v_{F-} + \frac{2Va\cos 2k_{F-}}{\pi} \end{bmatrix}, \tag{12}$$

$$\mathrm{v}_1 = \begin{bmatrix} \frac{1}{2}v_{F+} + \frac{(2-\cos 2k_{F+})Va}{2\pi} & \frac{(U+2V)a}{2\pi} \\ \frac{(U+2V)a}{2\pi} & \frac{1}{2}v_{F-} + \frac{(2-\cos 2k_{F-})Va}{2\pi} \end{bmatrix}. \tag{13}$$

These matrices are only valid for the first order of U and V. For zero field case $h = 0$, we have $k_{F+} = k_{F-}$ and $v_{F+} = v_{F-}$. In this case, matrices v_0 and v_1 commute, and there are eigenvectors for both matrices v_0 and v_1, corresponding to the separation of the charge and the spin parts. But, in general, for $h \neq 0$ we cannot diagonalize v_0 and v_1 simultaneously.

From the canonical equation, we obtain

$$\frac{\partial\phi_\sigma}{\partial t} = \sum_{\sigma'=\pm} v_{0\sigma\sigma'}\pi\Pi_{\sigma'} = \sum_{\sigma'=\pm} v_{0\sigma\sigma'}\frac{\partial\theta_{\sigma'}}{\partial x}, \quad \frac{\partial\theta_\sigma}{\partial t} = \sum_{\sigma'=\pm} v_{1\sigma\sigma'}\frac{\partial\phi_{\sigma'}}{\partial x}, \tag{14}$$

and we have the Lagrangian density

$$\mathcal{L} = \frac{1}{2\pi}\sum_{\sigma\sigma'}\left[(\mathrm{v}_0^{-1})_{\sigma\sigma'}\left(\frac{\partial\phi_\sigma}{\partial t}\right)\left(\frac{\partial\phi_{\sigma'}}{\partial t}\right) - v_{1\sigma\sigma'}\left(\frac{\partial\phi_\sigma}{\partial x}\right)\left(\frac{\partial\phi_{\sigma'}}{\partial x}\right)\right] + \sum_{\sigma=\pm}\frac{g_\sigma}{2\pi}\cos\big(2\sqrt{2}\phi_\sigma - 2(\pi - 2k_{F\sigma})x - 2k_{F\sigma}\big). \tag{15}$$

In the analysis, we use the following parameters for convenience:

$$w_0^2 = \det \mathrm{v}_0 = v_{0++}v_{0--} - v_{0+-}^2, \quad w_1^2 = \det \mathrm{v}_1 = v_{1++}v_{1--} - v_{1+-}^2, \quad w_2^2 = \mathrm{tr}\,\mathrm{v}_0\mathrm{v}_1 = v_{0++}v_{1++} + v_{0--}v_{1--} + 2v_{0+-}v_{1+-}. \tag{16}$$

4. Correlation functions of phase fields

Before considering the instability of the model (10), we calculate the correlation function for $g_\pm = 0$ in the thermodynamic limit $L \to \infty$. With the diagonalization of the two-component Hamiltonian (11), we can calculate the correlation function of the phase field [41–43]. But here we calculate it with the Lagrangian, aiming for an extension to the general multi-component case.

In order to stabilize the TL liquid, we need the condition such that the eigenvalues of v_0 and v_1 should be positive, so that

$$w_0^2 > 0, \qquad w_1^2 > 0, \tag{17}$$

and we assume this. If this condition is violated, the TL liquid is unstable. Due to the transformation (7), we consider the case $1/2 < n < 1$. In Euclidean space-time $t = -\mathrm{i}\tau$ (τ is the imaginary time), the free part of the Lagrangian density is written as

$$\mathcal{L}_0 = \frac{1}{2\pi}\sum_{\sigma\sigma'}\left[(\mathrm{v}_0^{-1})_{\sigma\sigma'}\left(\frac{\partial\phi_\sigma}{\partial t}\right)\left(\frac{\partial\phi_{\sigma'}}{\partial t}\right) + v_{1\sigma\sigma'}\left(\frac{\partial\phi_\sigma}{\partial x}\right)\left(\frac{\partial\phi_{\sigma'}}{\partial x}\right)\right]. \tag{18}$$

The correlation function of ϕ_σ for this free Lagrangian is given by

$$\begin{aligned}\Delta^{\phi\phi}(\tau_1-\tau_2, x_1-x_2) &= \begin{pmatrix} \langle\phi_+(\tau_1,x_1)\phi_+(\tau_2,x_2)\rangle_0 & \langle\phi_+(\tau_1,x_1)\phi_-(\tau_2,x_2)\rangle_0 \\ \langle\phi_-(\tau_1,x_1)\phi_+(\tau_2,x_2)\rangle_0 & \langle\phi_-(\tau_1,x_1)\phi_-(\tau_2,x_2)\rangle_0 \end{pmatrix} \\ &= \pi\int_{-\infty}^{\infty}\frac{\mathrm{d}\omega}{2\pi}\int_{-\pi/a}^{\pi/a}\frac{\mathrm{d}k}{2\pi}\mathrm{e}^{-\mathrm{i}\omega\tau_{12}+\mathrm{i}kx_{12}}\left[(\mathrm{v}_0^{-1})\omega^2 + \mathrm{v}_1\frac{2-2\cos ka}{a^2}\right]^{-1},\end{aligned} \tag{19}$$

where we used the notation $\tau_{12} = \tau_1 - \tau_2$ and $x_{12} = x_1 - x_2$. From the direct calculation, we obtain the explicit form of the correlation function:

$$\Delta^{\phi\phi}_{\sigma_1\sigma_2}(\tau_{12}, x_{12}) = \langle\phi_{\sigma_1}(\tau_1,x_1)\phi_{\sigma_2}(\tau_2,x_2)\rangle_0 = \tfrac{1}{2}\sum_{\bar\sigma=\pm}K^{\bar\sigma}_{\sigma_1\sigma_2}\Gamma_{\bar\sigma}(\tau_{12},x_{12}), \tag{20}$$

where

$$K^{\bar\sigma}_{\sigma_1\sigma_2} = \frac{\bar\sigma}{\sqrt{w_2^4 - 4w_0^2w_1^2}}[u_{\bar\sigma}v_{0\sigma_1\sigma_2} - w_0w_1u_{-\bar\sigma}(v_1^{-1})_{\sigma_1\sigma_2}] \tag{21}$$

$$u_{\bar\sigma} = \tfrac{1}{2}\left(\sqrt{w_2^2 + 2w_0w_1} + \bar\sigma\sqrt{w_2^2 - 2w_0w_1}\right) \tag{22}$$

$$\Gamma_{\bar\sigma}(\tau_{12},x_{12}) = \frac{2\pi}{u_{\bar\sigma}}\int_{-\infty}^{\infty}\frac{\mathrm{d}\omega}{2\pi}\int_{-\pi/a}^{\pi/a}\frac{\mathrm{d}k}{2\pi}\frac{\mathrm{e}^{-\mathrm{i}\omega\tau+\mathrm{i}kx}}{(\omega/u_{\bar\sigma})^2 + (2-2\cos ka)/a^2}. \tag{23}$$

Here, we assume $v_{0+}v_{1+} > v_{0-}v_{1-}$ corresponding to $k_{F+} > k_{F-}$ ($n_+ > n_-$).

Formally equation (20) is derived as follows. Equation (19) can be rewritten as

$$\Delta^{\phi\phi}(\tau_{12},x_{12}) = \pi\mathrm{v}_0^{1/2}\int_{-\infty}^{\infty}\frac{\mathrm{d}\omega}{2\pi}\int_{-\pi/a}^{\pi/a}\frac{\mathrm{d}k}{2\pi}\mathrm{e}^{-\mathrm{i}\omega\tau_{12}+\mathrm{i}kx_{12}}\left[\omega^2 + \mathrm{v}_0^{1/2}\mathrm{v}_1\mathrm{v}_0^{1/2}\frac{2-2\cos ka}{a^2}\right]^{-1}\mathrm{v}_0^{1/2}.$$

We diagonalize the matrix $\mathrm{v}_0^{1/2}\mathrm{v}_1\mathrm{v}_0^{1/2}$ with an orthogonal matrix O as

$$\mathrm{O}\mathrm{v}_0^{1/2}\mathrm{v}_1\mathrm{v}_0^{1/2}{}^{\mathrm{t}}\mathrm{O} = \mathrm{u}^2, \tag{24}$$

where ${}^{\mathrm{t}}\mathrm{O}$ is the transpose of the matrix O and u is a diagonal matrix diag (u_+, u_-) with positive elements $u_\pm > 0$. Then we have

$$\Delta^{\phi\phi}(\tau_{12},x_{12}) = \tfrac{1}{2}\mathrm{v}_0^{1/2}{}^{\mathrm{t}}\mathrm{O}\mathrm{u}^{-1}\Gamma(\tau_{12},x_{12})\mathrm{O}\mathrm{v}_0^{1/2}, \tag{25}$$

where

$$\Gamma(\tau_{12}, x_{12}) = \mathrm{diag}(\Gamma_+(\tau_{12}, x_{12}), \Gamma_-(\tau_{12}, x_{12})), \tag{26}$$

and $\Gamma_{\bar\sigma}$ is given by equation (23). Then we define the matrix $\mathrm{K}^{\bar\sigma}$, whose elements are given by

$$K^{\bar\sigma}_{\sigma_1\sigma_2} = (\mathrm{v}_0^{1/2\mathrm{t}}\mathrm{O})_{\sigma_1\bar\sigma} u_{\bar\sigma}^{-1} (\mathrm{O}\mathrm{v}_0^{1/2})_{\bar\sigma\sigma_2}, \tag{27}$$

and we obtain equation (20). We can also use another orthogonal transformation $\mathrm{Q}\mathrm{v}_1^{1/2}\mathrm{v}_0\mathrm{v}_1^{1/2\mathrm{t}}\mathrm{Q} = \mathrm{u}^2$, where Q is an orthogonal matrix, and we have $K^{\bar\sigma}_{\sigma_1\sigma_2} = (\mathrm{v}_1^{-1/2\mathrm{t}}\mathrm{Q})_{\sigma_1\bar\sigma} u_{\bar\sigma} (\mathrm{Q}\mathrm{v}_1^{-1/2})_{\bar\sigma\sigma_2}$.

The integral of the Green function $\Gamma_{\bar\sigma}$ (23) is infrared divergent and this comes from $\Gamma_{\bar\sigma}(0,0) = -(1/2)\log(\pi/2L)$, where L is the system size. But for the subtraction $G_{\bar\sigma}(\tau, x) = \Gamma_{\bar\sigma}(\tau, x) - \Gamma_{\bar\sigma}(0,0)$, the integral converges. For large $\sqrt{(u_{\bar\sigma}\tau)^2 + x^2}$ this function is approximated as

$$G_{\bar\sigma}(\tau, x) = -\ln \frac{\sqrt{(u_{\bar\sigma}\tau)^2 + x^2}}{r_0}, \tag{28}$$

where $r_0 = a/4\mathrm{e}^\gamma$ and $\gamma = 0.577\ldots$ is Euler's constant.

Using the canonical relation (14), we have the correlation function of θ_σ as

$$\Delta^{\theta\theta}_{\sigma_1\sigma_2}(\tau_{12}, x_{12}) = \langle\theta_{\sigma_1}(\tau_1, x_1)\theta_{\sigma_2}(\tau_2, x_2)\rangle_0 = \frac{1}{2}\sum_{\bar\sigma} K^{(-1)\bar\sigma}_{\sigma_1\sigma_2}\Gamma_{\bar\sigma}(\tau_{12}, x_{12}) \tag{29}$$

$$K^{(-1)\bar\sigma}_{\sigma_1\sigma_2} = (\mathrm{v}_0^{-1/2\mathrm{t}}\mathrm{O})_{\sigma_1\bar\sigma} u_{\bar\sigma} (\mathrm{O}\mathrm{v}_0^{-1/2})_{\bar\sigma\sigma_2}. \tag{30}$$

Matrices $\mathrm{K}^{(-1)\bar\sigma}$, whose elements are $K^{(-1)\bar\sigma}_{\sigma_1\sigma_2}$ and $\mathrm{K}^{\bar\sigma}$, are constructed from the velocity matrices v_0 and v_1, so that we express them as $\mathrm{K}^{(-1)\bar\sigma} = \mathrm{K}^{(-1)\bar\sigma}(\mathrm{v}_0, \mathrm{v}_1)$ and $\mathrm{K}^{\bar\sigma} = \mathrm{K}^{\bar\sigma}(\mathrm{v}_0, \mathrm{v}_1)$. Then from the canonical relation (14), we have a relation $\mathrm{K}^{(-1)\bar\sigma}(\mathrm{v}_0, \mathrm{v}_1) = \mathrm{K}^{\bar\sigma}(\mathrm{v}_1, \mathrm{v}_0)$. With equation (21), this gives the explicit form of $\mathrm{K}^{(-1)\bar\sigma}$. Conversely, we can express v_0 and v_1 with $u_{\bar\sigma}$, $\mathrm{K}^{\bar\sigma}$ and $\mathrm{K}^{(-1)\bar\sigma}$ as

$$\begin{aligned} \mathrm{v}_0 &= \sum_{\bar\sigma=\pm} u_{\bar\sigma}\mathrm{K}^{\bar\sigma}, & \mathrm{v}_0^{-1} &= \sum_{\bar\sigma=\pm} u_{\bar\sigma}^{-1}\mathrm{K}^{(-1)\bar\sigma}, \\ \mathrm{v}_1^{-1} &= \sum_{\bar\sigma=\pm} u_{\bar\sigma}^{-1}\mathrm{K}^{\bar\sigma}, & \mathrm{v}_1 &= \sum_{\bar\sigma=\pm} u_{\bar\sigma}\mathrm{K}^{(-1)\bar\sigma}. \end{aligned} \tag{31}$$

From equations (27) and (30), matrices $\mathrm{K}^{\bar\sigma}$ and $\mathrm{K}^{(-1)\bar\sigma}$ satisfy the relation

$$\sum_{\bar\sigma=\pm}\mathrm{K}^{\bar\sigma}\mathrm{K}^{(-1)\bar\sigma} = \sum_{\bar\sigma=\pm}\mathrm{K}^{(-1)\bar\sigma}\mathrm{K}^{\bar\sigma} = \mathbf{1}$$

(where $\mathbf{1}$ is the unit matrix) and

$$\sum_{\sigma_1,\sigma_2=\pm} K^{\bar\sigma}_{\sigma_1\sigma_2} K^{(-1)\bar\sigma'}_{\sigma_2\sigma_1} = \sum_{\sigma_1,\sigma_2=\pm} K^{(-1)\bar\sigma}_{\sigma_1\sigma_2} K^{\bar\sigma'}_{\sigma_2\sigma_1} = \delta_{\bar\sigma\bar\sigma'}. \tag{32}$$

Lastly, using equation (14) the correlation function between ϕ and θ is calculated as

$$\Delta^{\phi\theta}_{\sigma_1\sigma_2}(\tau_{12}, x_{12}) = \langle\phi_{\sigma_1}(\tau_1, x_1)\theta_{\sigma_2}(\tau_2, x_2)\rangle_0 = -\frac{\mathrm{i}}{2}\sum_{\bar\sigma=\pm} L^{\bar\sigma}_{\sigma_1\sigma_2}\arg(u_{\bar\sigma}\tau_{12} + \mathrm{i}x_{12}), \tag{33}$$

$$L^{\bar\sigma}_{\sigma_1\sigma_2} = (\mathrm{v}_0^{1/2\mathrm{t}}\mathrm{O})_{\sigma_1\bar\sigma}(\mathrm{O}\mathrm{v}_0^{-1/2})_{\bar\sigma\sigma_2}. \tag{34}$$

The matrix $\mathrm{L}^{\bar\sigma}$, whose elements are $L^{\bar\sigma}_{\sigma_1\sigma_2}$, satisfies $\mathrm{L}^{\bar\sigma} = \mathrm{K}^{\bar\sigma}\mathrm{K}^{(-1)\bar\sigma}$ and $\sum_{\bar\sigma}\mathrm{L}^{\bar\sigma} = \mathbf{1}$. Determinants of the matrices $\mathrm{K}^{\bar\sigma}$, $\mathrm{K}^{(-1)\bar\sigma}$ and $\mathrm{L}^{\bar\sigma}$ are zero.

According to [14, 15, 21, 23, 24], the low energy property of the system is shown as two independent TL liquids. Let us derive such expressions in our formulation. We define new fields:

$$\begin{aligned}\varphi_{L\bar{\sigma}} &= \sum_{\sigma=\pm}[u_{\bar{\sigma}}^{1/2}(\mathrm{Ov}_0^{-1/2})_{\bar{\sigma}\sigma}\phi_\sigma + u_{\bar{\sigma}}^{-1/2}(\mathrm{Ov}_0^{1/2})_{\bar{\sigma}\sigma}\theta_\sigma],\\ \varphi_{R\bar{\sigma}} &= \sum_{\sigma=\pm}[-u_{\bar{\sigma}}^{1/2}(\mathrm{Ov}_0^{-1/2})_{\bar{\sigma}\sigma}\phi_\sigma + u_{\bar{\sigma}}^{-1/2}(\mathrm{Ov}_0^{1/2})_{\bar{\sigma}\sigma}\theta_\sigma].\end{aligned} \tag{35}$$

Then from the relation $[\phi_\sigma(x), \theta_{\sigma'}(x')] = -\delta_{\sigma,\sigma'}\mathrm{i}(\pi/2)\,\mathrm{sgn}\,(x-x')$, we can see

$$\begin{aligned}&[\varphi_{L\bar{\sigma}}(x), \varphi_{L\bar{\sigma}'}(x')] = -\mathrm{i}\pi\delta_{\bar{\sigma}\bar{\sigma}'}\mathrm{sgn}\,(x-x'),\\ &[\varphi_{R\bar{\sigma}}(x), \varphi_{R\bar{\sigma}'}(x')] = \mathrm{i}\pi\delta_{\bar{\sigma}\bar{\sigma}'}\mathrm{sgn}\,(x-x'),\\ &[\varphi_{L\bar{\sigma}}(x), \varphi_{R\bar{\sigma}'}(x')] = 0.\end{aligned}$$

Thus these fields are independent. Then we define the current operator as

$$J_{L\bar{\sigma}} = \mathrm{i}\frac{1}{2}\left(\frac{1}{u_{\bar{\sigma}}}\frac{\partial}{\partial\tau} - \mathrm{i}\frac{\partial}{\partial x}\right)\varphi_{L\bar{\sigma}}, \qquad J_{R\bar{\sigma}} = \mathrm{i}\frac{1}{2}\left(\frac{1}{u_{\bar{\sigma}}}\frac{\partial}{\partial\tau} + \mathrm{i}\frac{\partial}{\partial x}\right)\varphi_{R\bar{\sigma}}, \tag{36}$$

whose correlation functions are

$$\begin{aligned}\langle J_{L\bar{\sigma}}(\tau_1, x_1)J_{L\bar{\sigma}'}(\tau_2, x_2)\rangle_0 &= \delta_{\bar{\sigma}\bar{\sigma}'}\frac{1}{(u_{\bar{\sigma}}\tau_{12} + \mathrm{i}x_{12})^2},\\ \langle J_{L\bar{\sigma}}(\tau_1, x_1)J_{R\bar{\sigma}'}(\tau_2, x_2)\rangle_0 &= 0,\\ \langle J_{R\bar{\sigma}}(\tau_1, x_1)J_{R\bar{\sigma}'}(\tau_2, x_2)\rangle_0 &= \delta_{\bar{\sigma}\bar{\sigma}'}\frac{1}{(u_{\bar{\sigma}}\tau_{12} - \mathrm{i}x_{12})^2}.\end{aligned} \tag{37}$$

Using these operators, the free Lagrangian (18) and Hamiltonian densities (11) can be written as

$$\mathcal{L}_0 = -\frac{1}{2\pi}\sum_{\bar{\sigma}=\pm}u_{\bar{\sigma}}J_{L\bar{\sigma}}J_{R\bar{\sigma}}, \qquad \mathcal{H}_0 = \frac{1}{2\pi}\sum_{\bar{\sigma}=\pm}\frac{u_{\bar{\sigma}}}{2}(J_{L\bar{\sigma}}J_{L\bar{\sigma}} + J_{R\bar{\sigma}}J_{R\bar{\sigma}}). \tag{38}$$

Thus these are composed of two independent parts of $\bar{\sigma} = \pm$. $u_{\bar{\sigma}}$ is the sound velocity of each part and these are equivalent to $v_{F\pm}$ for the non-interacting case $U = V = 0$. Due to the condition (17), sound velocities $u_\pm$ are real.

The extension of the above procedure to general multi-component TL liquids [22, 24] is straightforward. For the N-component case, we have $N \times N$ velocity matrices v_0 and v_1, and we obtain N sound velocities $u_{\bar{\sigma}}$. Comparing the scaling dimensions of scaling operators, we obtain the dressed charge matrix of [22] as

$$\mathrm{Z} = \frac{1}{\sqrt{2}}\mathrm{v}_0^{1/2\mathrm{t}}\mathrm{O}\mathrm{u}^{-1/2}\left(= \frac{1}{\sqrt{2}}\mathrm{v}_1^{-1/2\mathrm{t}}\mathrm{Q}\mathrm{u}^{1/2}\right). \tag{39}$$

Elements of matrices $\mathrm{K}^{\bar{\sigma}}$, $\mathrm{K}^{(-1)\bar{\sigma}}$ and $\mathrm{L}^{\bar{\sigma}}$ are described as

$$\begin{aligned}K_{\sigma_1\sigma_2}^{\bar{\sigma}} &= 2Z_{\sigma_1\bar{\sigma}}({}^{\mathrm{t}}\mathrm{Z})_{\bar{\sigma}\sigma_2},\\ K_{\sigma_1\sigma_2}^{(-1)\bar{\sigma}} &= \tfrac{1}{2}({}^{\mathrm{t}}\mathrm{Z}^{-1})_{\sigma_1\bar{\sigma}}(\mathrm{Z}^{-1})_{\bar{\sigma}\sigma_2},\\ L_{\sigma_1\sigma_2}^{\bar{\sigma}} &= Z_{\sigma_1\bar{\sigma}}(\mathrm{Z}^{-1})_{\bar{\sigma}\sigma_2}, \qquad \sigma_1, \sigma_2, \bar{\sigma} = 1, \ldots, N.\end{aligned} \tag{40}$$

We define the matrices u, $\mathrm{K}^{\bar{\sigma}}$, $\mathrm{K}^{(-1)\bar{\sigma}}$ and $\mathrm{L}^{\bar{\sigma}}$ from v_0 and v_1. Among $N(N+1)$ independent elements of v_0 and v_1, we have N sound velocities $u_{\bar{\sigma}}$ and the number of independent elements of the matrices $\mathrm{K}^{\bar{\sigma}}$, $\mathrm{K}^{(-1)\bar{\sigma}}$ and $\mathrm{L}^{\bar{\sigma}}$ is N^2, the dimension of the matrix Z. Equations (35) are written as

$$\begin{aligned}\varphi_{L\bar{\sigma}} &= \sum_\sigma\left(\frac{1}{\sqrt{2}}(\mathrm{Z}^{-1})_{\bar{\sigma}\sigma}\phi_\sigma + \sqrt{2}({}^{\mathrm{t}}\mathrm{Z})_{\bar{\sigma}\sigma}\theta_\sigma\right),\\ \varphi_{R\bar{\sigma}} &= \sum_\sigma\left(-\frac{1}{\sqrt{2}}(\mathrm{Z}^{-1})_{\bar{\sigma}\sigma}\phi_\sigma + \sqrt{2}({}^{\mathrm{t}}\mathrm{Z})_{\bar{\sigma}\sigma}\theta_\sigma\right).\end{aligned} \tag{41}$$

For the Hubbard model, another type of dressed charge matrix is defined in relation to the charge and spin freedoms. For the matrix of [14, 21], we obtain

$$Z = \begin{pmatrix} Z_{cc} & Z_{cs} \\ Z_{sc} & Z_{ss} \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} \mathrm{v}_0^{1/2\mathrm{t}} \mathrm{O} \mathrm{u}^{-1/2}. \tag{42}$$

5. Instability

Using equations (20), (29), and (33), we can calculate the correlation functions of several operators. The element of matrices $\mathrm{K}^{\bar{\sigma}}$ and $\mathrm{K}^{(-1)\bar{\sigma}}$ gives exponents of them. In the calculation, we must take into account the charge neutrality condition of the Coulomb gas [44] due to the infrared divergence of $\Gamma_{\bar{\sigma}}(0, 0)$. From equation (20), the correlation function of the operator $\cos 2\sqrt{2}\phi_+$, which generates the umklapp scattering between two electrons with spin + for $n_+ = 1/2$ (see equations (10) and (15)), is given by

$$\begin{aligned} 2\langle \cos 2\sqrt{2}\phi_+(\tau_1, x_1) \cos 2\sqrt{2}\phi_+(\tau_2, x_2)\rangle_0 &= \exp\left(4 \sum_{\bar{\sigma}=\pm} K^{\bar{\sigma}}_{++} G_{\bar{\sigma}}(\tau_{12}, x_{12})\right) \\ &= \left(\frac{r_0^2}{(u_+\tau_{12})^2 + x_{12}^2}\right)^{2K^+_{++}} \left(\frac{r_0^2}{(u_-\tau_{12})^2 + x_{12}^2}\right)^{2K^-_{++}}. \end{aligned} \tag{43}$$

Thus the scaling dimension of the operator $\cos 2\sqrt{2}\phi_+$ is

$$x = 2K^+_{++} + 2K^-_{++} = 2\frac{v_{0++} + w_0 w_1 (v_1^{-1})_{++}}{\sqrt{w_2^2 + 2w_0 w_1}}. \tag{44}$$

From the scaling argument, when the scaling dimension x is greater than 2 and $k_{F+} = \pi/2$, the operator $\cos 2\sqrt{2}\phi_+$ is irrelevant and the coupling g_+ in equation (10) is renormalized to zero. The other independent scaling parameters u_+, u_-, K^+_{++}, K^+_{+-}, K^-_{--}, K^-_{+-} are renormalized to finite values. Thus the low energy property of the system is of two TL liquids.

When $x < 2$ and $k_{F+} = \pi/2$, the operator $\cos 2\sqrt{2}\phi_+$ is relevant. In this case, the coupling g_+ is renormalized to infinity and elements $K^{\bar{\sigma}}_{+,+}$ and $K^{\bar{\sigma}}_{+,-}$ ($\bar{\sigma} = \pm$) are renormalized to zero. The phase field ϕ_+ is locked as $\phi_+ = \pi/2\sqrt{2}$ or $3\pi/2\sqrt{2}$. Since the density operator of electrons with spin $\sigma = +$ is written as

$$c^\dagger_{x+} c_{x+} = \frac{1}{\pi a} \sin(\sqrt{2}\phi_+ - 2k_{F+}x) - \frac{1}{\sqrt{2}\pi} \frac{\partial \phi_+}{\partial x} + n_+, \tag{45}$$

electrons with $\sigma = +$ occupy every other site. There exists an excitation gap for these electrons. Electrons with $\sigma = -$ feel a potential from this crystal of $\sigma = +$ electrons. But in our considered order of U and V, there is no mechanism for generating a gap for $\sigma = -$ electrons because of the incommensurability $k_{F-} < \pi/2$. The low energy physics is described by the one-component TL liquid with the freedom of electrons with spin $\sigma = -$. Due to a gap for $\sigma = +$ electrons, there is a magnetic gap, and this means that there can exist a magnetization plateau in the magnetization curve at $m = (1 - n)/n$. Although there is a magnetic gap, the system is not an insulator but a conductor, relating to $\sigma = -$ fermions. This instability is in the context of the generalized Luttinger theorem [37].

Using the weak coupling parameter (12) and (13), we calculate the line $x = 2$ of equation (44) for the system with $n_+ = 0.5$, $n_- = 0.2$ and with $n_+ = 0.5$, $n_- = 0.3$ in figure 1. We can see that, for large V, the above instability can appear. Since equations (12) and (13) are valid for small U and V, figure 1 shows only the tendency of the boundary.

In the previous section, we assumed the condition (17) which is needed for the stability of the TL liquid. Here we consider the limit $w_{0,1} \to +0$. For the one-component case,

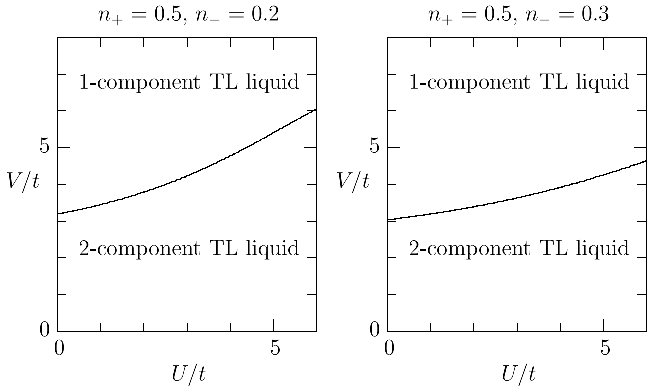

Figure 1. Boundary between the one-component and two-component TL liquids for $n_+ = 0.5$, $n_- = 0.2$ and for $n_+ = 0.5$, $n_- = 0.3$.

an instability of phase separation appears in the limit $uK \to$ finite and $u/K \to 0$ [45]. Let us consider the case that one eigenvalue of v_1 approaches to zero. In general, w_1^2 goes to zero linearly for the coupling parameters U and V. From equation (22), near $w_1 = 0$ we have

$$u_+ \approx w_2, \qquad u_- \approx \frac{w_0 w_1}{w_2}. \tag{46}$$

Thus u_- goes to zero as $u_- \propto \sqrt{w_1^2}$. From equation (21) we can also see that $K^-_{\sigma_1\sigma_2} \propto 1/\sqrt{w_1^2}$, $K^{(-1)-}_{\sigma_1\sigma_2} \propto \sqrt{w_1^2}$. These vanishing and divergence behaviours are the same as in the one-component case [45]. When w_1^2 becomes negative, phase separation into two regions with different densities of electrons appears as for the one-component case. We can expect the appearance of the phase-separated state in negative large V regions. The magnetization behaviours between the normal two-component TL liquid and the phase-separated system are different. Thus we can expect that there exists a cusp in the magnetization curve at the boundary of the normal and phase-separated regions.

Next let us look at the limit $w_0^2 \to +0$. In this case, with the same calculation for the limit $w_1^2 \to 0$, we obtain $u_- \propto \sqrt{w_0^2}$, $K^-_{\sigma,\sigma'} \propto \sqrt{w_0^2}$ and $K^{(-1)-}_{\sigma,\sigma'} \propto 1/\sqrt{w_0^2}$. If w_0^2 were negative, numbers of the left moving and right moving fermions would be different in the ground state, and spontaneous electron currents could appear. I think that the limit $w_0^2 \to +0$ would not be realized for finite U and V in the model under consideration.

6. Summary and discussion

We studied the instability of the 1D extended Hubbard model in a magnetic field. For this model, an instability can appear from the umklapp scattering process between electrons with the same spin. Considering the commensurability from the density n_+ and n_-, we saw that this instability can occur for the magnetization $m = (1-n)/n$ for $1/2 < n < 1$ and $m = (n-1)/(2-n)$ for $1 < n < 3/2$. A magnetization plateau can exist in the magnetization curve at this magnetization. In the plateau region, the system is not an insulator but metallic.

In the analysis, we used the bosonization calculation. The $U(1) \times U(1)$ symmetry of the system is crucial for the calculation in section 4. In this context, we calculated the correlation function of the phase field for the free field case and derived sound velocities $u_{\bar{\sigma}}$ and matrices

$K^{\bar{\sigma}}$ and $K^{(-1)\bar{\sigma}}$ which give critical exponents. This calculation gives the correspondence between the bosonization approach and the BA results. The instability from the umklapp scattering process occurs when $k_{F\sigma} = \pi/2$ and $x = \sum_{\bar{\sigma}} K^{\bar{\sigma}}_{\sigma\sigma} = 2$ (with renormalized $K^{\bar{\sigma}}_{\sigma\sigma}$). Considering the generation of an excitation gap from the critical gapless system, the quantum phase transition should be of the Berezinskii–Kosterlitz–Thouless type [46–48].

In the numerical approach for the finite-size system, we can calculate the matrices of the velocity v_0 and v_1, and the sound velocity $u_{\bar{\sigma}}$ from the excitation energy [8, 14, 15, 21]. (Using equation (22), we can check the validity of the TL liquid assumption.) Using the finite-size spectrum, we can calculate the scaling dimension (44) and can estimate crude points where $x = 2$.

In the above discussion, we assumed the condition (17), which ensures the stability of TL liquid for $g_\sigma = 0$. When these conditions are violated, we can no longer apply the above analysis to give the density n and magnetization m. In this case, we expect the appearance of the phase separation. For zero magnetic field $h = 0$, the phase-separated phase in negative V regions has been reported [28, 29, 33, 34]. We can expect that this phase remains even for systems with some finite (or saturated) magnetization.

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